A Novel Error Performance Analysis Methodology for OFDM with Index Modulation

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***Abstract*—Orthogonal frequency-division multiplexing with index modulation (OFDM-IM) has become a high-profile candidate for the modulation scheme in next generation networks, and a large number of works have been published to analyze it in recent years. Error performance is one of the most important and interesting aspects of OFDM-IM, which has been analyzed in most works. However, most of them employ two key approximations to derive the closed-form expressions of block error rate (BLER) and/or average bit error rate (BER). The first one is to utilize the union bound assisted by the pairwise error probability (PEP) analysis, and the second one is to employ an exponential approximation of Q-function. In this letter, we apply Craig's formula to analyze the error performance for OFDM-IM in place of the exponential approximation. Because Craig's formula is the exact expression of Q-function, the accuracy of analytical results regarding error performance for OFDM-IM can be improved. We examine the proposed error performance analysis methodology based on Craig's formula by investigating both average BLER and BER.**

***Index Terms*—Orthogonal frequency-division multiplexing with index modulation (OFDM-IM), error performance analysis, performance approximation, Q-function, Craig's formula.**

I. Introduction

ORTHOGONAL frequency-division multiplexing with index modulation (OFDM-IM) becomes one of the most promising modulation candidates in next generation networks, and has been exhaustively studied in recent years [1]. OFDM-IM employs an additional modulation dimension termed the *index* dimension in addition to the amplitude and phase of constellation symbol to convey extra information bits by subcarrier activation patterns (SAPs). In this way, a better performance would be attained by setting proper system configurations [2]–[4]. Because OFDM-IM applies a unique modulation procedure to encode incoming bits, one of the most important aspects of OFDM-IM is the error performance, which can be characterized by average block error rate (BLER) and/or bit error rate (BER). Therefore, either of them is investigated in most works related to OFDM-IM. However, in order to simplify the derivations and obtain closed-form expressions, the error performance analysis methodology in most existing works is based on two approximations stemming from the original work of the plain OFDM-IM presented in [5]. The first one is to utilize the union bound assisted by the pairwise error probability (PEP) analysis. The second one is to employ an exponential approximation of Q-function.

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Because of the employments of aforementioned two approximations, the accuracy of the analytical results of error performance for OFDM-IM loses and a gap exists between analytical and numerical results, especially for average BER. To enhance the analytical accuracy and explore the mechanism that causes the gap between analytical and numerical results, we abandon the exponential approximation of Q-function and adopt its exact expression in the form of Craig's formula [6]. By the Fubini–Tonelli theorem [7], the closed-form expressions of both average BLER and BER are obtained by this new error performance analysis methodology, and therefore easy to perform further analysis. As a result, the analytical accuracy will only lose due to the employed union bound with PEP analysis. Consequently, more accurate analytical results are achievable, and the impacts of both approximations on the gap between numerical and analytical results can be clarified. Numerical results generated by Monte Carlo simulations verify the feasibility of the proposed methodology.

Note that, we only take the plain OFDM-IM as an example to illustrate the feasibility of the proposed methodology based on Craig's formula, but the derived results can be easily expanded to other more advanced cases associated with OFDM-IM. Moreover, the proposed methodology can also be easily tailored to apply to other 'sibling' schemes of OFDM-IM, e.g. subcarrier-index modulation (SIM) OFDM and OFDM with subcarrier number modulation (SNM) etc.

II. System Model

The system model and assumptions adopted in this letter are exactly the same as those stipulated in [5]. Meanwhile, to simplify the discussion and dedicate to the new error performance analysis methodology of error performance, we focus on only one single group consisting of N subcarriers with independent and identically distributed (i.i.d.) fading generated by inverse fast Fourier transform (IFFT) and interleaved grouping [8], among which K subcarriers are activated to form a SAP, so as to modulate $\lfloor\log_2\binom{N}{K}\rfloor$-bit heading stream, where $\lfloor\cdot\rfloor$ denotes the floor function. To be simple, we follow the look-up table mapping relation specified in [5]. Furthermore, a sufficiently long cyclic prefix (CP) and perfect synchronization in both time and frequency domains are supposed in order to eliminate inter-symbol interference (ISI) and inter-channel interference (ICI). In this way, the OFDM-IM system can be considered and analyzed on a subcarrier basis. For each active subcarrier, conventional amplitude-phase modulation (APM) is applied and we adopt M-ary phase-shift keying (M-PSK) in this letter, owing to its constant-envelope attribute. Consequently, $K\log_2(M)$-bit subsequent stream is modulated to be data constellation symbols conveyed on K active subcarriers. Hence, $B = \lfloor \log_2 \binom{N}{K} \rfloor + K\log_2(M)$ is the length of the entire bit stream and all these incoming bits are supposed to be equiprobable. For convenience, we denote the set of all N subcarriers as $\mathcal{N} = \{1, 2, \ldots, N\}$ and set of K active subcarriers as $\mathcal{K}(t)$, where t is the index of the chosen OFDM block for transmission depending on the entire B-bit stream.

By IFFT and interleaved grouping, we can write the OFDM block for transmission as $\mathbf{x}(t) = [x(t,1), x(t,2), \ldots, x(t,N)]^T \in \mathbb{C}^{N\times 1}$, where $x(t,n) = \begin{cases} \chi_n, & \text{if } n \in \mathcal{K}(t) \\ 0, & \text{otherwise} \end{cases}$ and χ_n represents the M-ary data constellation symbol conveyed on the nth active subcarrier, which is normalized by $\chi_n \chi_n^* = 1$ for simplicity. More details of the transmission module of OFDM-IM can be found in [5].

Propagating through i.i.d. Rayleigh fading channels over all subcarriers, the received OFDM block can be written as $\mathbf{y}(t) = \sqrt{\frac{P_t}{K}}\mathbf{H}\mathbf{x}(t) + \mathbf{w} \in \mathbb{C}^{N\times 1}$, where P_t is the transmit power for the OFDM block, which is uniformly distributed over K active subcarriers; $\mathbf{H} = \mathrm{diag}\{h(1), h(2), \ldots, h(N)\}$ is the channel state matrix (CSM) consisting of the channel coefficients pertaining to N subchannels $h(n)$ that characterize the channel state information (CSI), and the probability density function (PDF) and cumulative distribution function (CDF) with respect to the exponentially distributed channel power gain $G(n) = |h(n)|^2$ with mean μ are given by $f_g(\xi) = \frac{1}{\mu}\exp\left(-\frac{\xi}{\mu}\right)$ and $F_g(\xi) = 1 - \exp\left(-\frac{\xi}{\mu}\right)$ for Rayleigh fading channel model; $\mathbf{w} = [w(1), w(2), \ldots, w(N)]^T$ is the vector of additive white Gaussian noise (AWGN) at the receiver, and $w(n) \sim \mathcal{CN}(0, N_0)$ is the random noise term on the nth subcarrier with the average noise power N_0.

To provide the optimal estimation and serve as the benchmark for other sub-optimal estimation schemes, we apply the maximum-likelihood (ML) estimation at the OFDM receiver, and the estimated OFDM block is produced by

$$\hat{\mathbf{x}}(\hat{t}) = \underset{\dot{\mathbf{x}}(\dot{t})\in\mathcal{X}}{\arg\min} \left\| \mathbf{y}(t) - \sqrt{\tfrac{P_t}{K}}\mathbf{H}\dot{\mathbf{x}}(\dot{t}) \right\|_F, \tag{1}$$

where $\left\|\cdot\right\|_F$ denotes the Frobenius norm of the enclosed matrix/vector and $\mathcal{X}$ is the set of all legitimate OFDM blocks for transmission by OFDM-IM. In this letter, we assume a slow fading scenario so that CSM $\mathbf{H}$ is supposed to be perfectly estimated at the receiver.

III. Error Performance Analysis

Because the information bits are modulated by both indices of active subcarriers and the data constellation symbols conveyed on all active subcarriers, the entire OFDM block should be estimated holistically [9]. Therefore, to characterize the error performance for OFDM-IM systems, one shall consider in a block level and/or in a bit level (only if the length of entire bit stream is invariant), which are characterized by average BLER and/or BER, respectively. Both measures are also highly connected and would be mutually transferable by a set of simple steps. In this section, we illustrate how to derive both in closed form by the proposed methodology based on Craig's formula and the Fubini–Tonelli theorem. Meanwhile, the conventional methodology relying on the exponential approximation of Q-function is also briefly discussed for comparison purposes.

A. Definitions of Average BLER and BER

1) Average BLER: Because the average BLER is more fundamental and average BER can be easily derived concomitantly, we first focus on the derivation of average BLER. In general without involving any specific analytical methodology, we can express the conditional BLER on CSM $\mathbf{H}$ to be $P_e(\mathbf{x}(t)|\mathbf{H}) = \mathbb{P}\left\{\hat{\mathbf{x}}(\hat{t}) \neq \mathbf{x}(t)|\mathbf{H}\right\}$, where $\mathbb{P}\{\cdot\}$ denotes the probability of the random event enclosed. Following this, we attain the unconditional BLER by averaging $P_e(\mathbf{x}(t)|\mathbf{H})$ over CSM $\mathbf{H}$ and have $P_e(\mathbf{x}(t)) = \underset{\mathbf{H}}{\mathbb{E}}\{P_e(\mathbf{x}(t)|\mathbf{H})\}$ for the OFDM block $\mathbf{x}(t)$, where $\mathbb{E}\{\cdot\}$ denotes the expected value of the enclosed. To consider different OFDM blocks, we average it over $\mathbf{x}(t) \in \mathcal{X}$, which yields the average BLER $\bar{P}_e = \underset{\mathbf{x}(t)\in\mathcal{X}}{\mathbb{E}}\{P_e(\mathbf{x}(t))\}$.

2) Average BER: To characterize the error performance in a bit level, one can resort to the average BER. Likewise, the conditional BER on CSM $\mathbf{H}$ can be expressed by[1] $P_b(\mathbf{x}(t)|\mathbf{H}) = \frac{1}{B}\mathbb{P}\left\{\hat{\mathbf{x}}(\hat{t}) \neq \mathbf{x}(t)|\mathbf{H}\right\}\delta(\hat{\mathbf{x}}(\hat{t}) \neq \mathbf{x}(t))$, where $\delta(\hat{\mathbf{x}}(\hat{t}) \neq \mathbf{x}(t))$ represents the number of bit errors when the originally transmitted OFDM block $\mathbf{x}(t)$ is erroneously estimated. Again, we ditto average it over CSM $\mathbf{H}$ and obtain $P_b(\mathbf{x}(t)) = \underset{\mathbf{H}}{\mathbb{E}}\{P_b(\mathbf{x}(t)|\mathbf{H})\}$ for the OFDM block $\mathbf{x}(t)$. Similarly, the average BER is produced by the following relation: $\bar{P}_b = \underset{\mathbf{x}(t)\in\mathcal{X}}{\mathbb{E}}\{P_b(\mathbf{x}(t))\}$.

B. Derivations of Average BLER and BER

1) Average BLER: To derive the average BLER, we first dedicate to the derivation of its basic element, i.e. the conditional BLER on CSM $\mathbf{H}$ $P_e(\mathbf{x}(t)|\mathbf{H})$. We can employ the classic union bound approach assisted by PEP analysis and approximate $P_e(\mathbf{x}(t)|\mathbf{H})$ to be

$$P_e(\mathbf{x}(t)|\mathbf{H}) \approx \sum_{\hat{\mathbf{x}}(\hat{t})\neq\mathbf{x}(t)} P_e\left(\mathbf{x}(t) \to \hat{\mathbf{x}}(\hat{t})|\mathbf{H}\right), \tag{2}$$

where $P_e\left(\mathbf{x}(t) \to \hat{\mathbf{x}}(\hat{t})|\mathbf{H}\right)$ represents the conditional PEP on CSM $\mathbf{H}$ that the originally transmitted OFDM block $\mathbf{x}(t)$ is erroneously estimated to $\hat{\mathbf{x}}(\hat{t})$. By the ML estimation scheme specified in (1), the conditional PEP $P_e\left(\mathbf{x}(t) \to \hat{\mathbf{x}}(\hat{t})|\mathbf{H}\right)$ can be written by Q-function $Q(x) = \frac{1}{2\pi}\int_x^{\infty}\exp\left(-u^2/2\right)\mathrm{d}u$ as

$$\begin{aligned} P_e\left(\mathbf{x}(t) \to \hat{\mathbf{x}}(\hat{t})|\mathbf{H}\right) &= Q\left(\sqrt{\frac{P_t}{KN_0}\left\|\mathbf{H}\left(\mathbf{x}(t) - \hat{\mathbf{x}}(\hat{t})\right)\right\|_F^2}\right) \\ &= Q\left(\sqrt{\frac{P_t}{KN_0}\sum_{n=1}^{N} G(n)\Delta(n,t,\hat{t})}\right), \end{aligned} \tag{3}$$

where $\Delta(n,t,\hat{t}) = \left|x(t,n) - x(\hat{t},n)\right|^2$ for simplicity.

[1] This expression of conditional BER is only applicable for modulation schemes with fixed transmission rate B in bit per channel use (bpcu).

By the original definition of Q-function, the argument is the lower limit of the interval of an integral, and thus it is demanding to carry out advanced analysis. To ease advanced analysis, an exponential approximation of Q-function $Q(x) \approx \frac{1}{12}\exp\left(-\frac{x^2}{2}\right) + \frac{1}{4}\exp\left(-\frac{2x^2}{3}\right)$ is adopted in the initial work of the plain OFDM-IM presented in [5], which gives a simple, neat and closed-form expression of the average error rate in both bit and block level. Therefore, most works analyzing error performance for OFDM-IM follow this approximation. However, because of this approximation as well as the union bound approach, the analytical accuracy reduces twice. Moreover, the simultaneous implementations of two approximations blur the mechanism causing the gap between analytical and numerical results. Consequently, we adopt Craig's formula $Q(x) = \frac{1}{\pi}\int_0^{\pi} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) \mathrm{d}\theta$ to circumvent the approximation of Q-function [6], and aim at obtaining a more accurate result as well as discovering the mechanism causing the gap between analytical and numerical results of the error performance for OFDM-IM. Accordingly, (3) can be rewritten as

$$\begin{aligned} &P_e\left(\mathbf{x}(t) \to \hat{\mathbf{x}}(\hat{t})|\mathbf{H}\right) \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{P_t}{2KN_0\sin^2\theta}\sum_{n=1}^{N} G(n)\Delta(n,t,\hat{t})\right) \mathrm{d}\theta \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{n=1}^{N} \exp\left(-\frac{P_t G(n)\Delta(n,t,\hat{t})}{2KN_0\sin^2\theta}\right) \mathrm{d}\theta. \end{aligned} \tag{4}$$

Following this, The unconditional PEP $P_e\left(\mathbf{x}(t) \to \hat{\mathbf{x}}(\hat{t})\right)$ can be determined in (5), where (a) is valid according to the Fubini–Tonelli theorem that allows swapping order of integration [7]; (b) is derived by the independence among subcarriers by the system model assumed in this paper; (c) is derived by partial fraction decomposition and the exchangeability between integral and summation operations; $\tau(n,t,\hat{t}) = \frac{P_t\mu\Delta(n,t,\hat{t})}{2KN_0}$; $\{\alpha(n,t,\hat{t})\}_{n=1}^{N}$ is a set of constant coefficients depending only on $\{\tau(n,t,\hat{t})\}_{n=1}^{N}$ and is irrelevant to θ. More specifically, $\{\alpha(n,t,\hat{t})\}_{n=1}^{N}$ can be easily solved by the equation set infra through linear programming[2]:

$$\begin{cases} \sum\limits_{n=1}^{N} \alpha(n,t,\hat{t}) = 1 \\ \sum\limits_{n=1}^{N} \left(\alpha(n,t,\hat{t}) \sum\limits_{\nu\in\mathcal{V}(\mathcal{N}\setminus\{n\},1)} \prod\limits_{u\in\nu} \tau(u,t,\hat{t})\right) = 0 \\ \sum\limits_{n=1}^{N} \left(\alpha(n,t,\hat{t}) \sum\limits_{\nu\in\mathcal{V}(\mathcal{N}\setminus\{n\},2)} \prod\limits_{u\in\nu} \tau(u,t,\hat{t})\right) = 0 \\ \qquad\vdots \\ \sum\limits_{n=1}^{N} \left(\alpha(n,t,\hat{t}) \sum\limits_{\nu\in\mathcal{V}(\mathcal{N}\setminus\{n\},N-1)} \prod\limits_{u\in\nu} \tau(u,t,\hat{t})\right) = 0 \end{cases} \tag{6}$$

where $\mathcal{V}(\mathcal{S}, i)$ denotes the set of all i-element subsets of a finite set $\mathcal{S}$. For example, if $\mathcal{S} = \{1,2,3,4\}$, then $\mathcal{V}(\mathcal{S},2) = \{\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\}\}$.

[2]For large N, a fast recursive algorithm would be used, so as to have the linear or quasi-linear computational complexity [10].

For comparison purposes, we also approximate $P_e\left(\mathbf{x}(t) \to \hat{\mathbf{x}}(\hat{t})\right)$ by the conventional methodology using the exponential approximation of Q-function as follows:

$$P_e\left(\mathbf{x}(t) \to \hat{\mathbf{x}}(\hat{t})\right) \approx \sum_{i=1}^{2}\left[\rho_i \prod_{n=1}^{N} \left(1 + 2\eta_i\tau(n,t,\hat{t})\right)^{-1}\right], \tag{7}$$

where $\{\rho_1, \rho_2\} = \{1/12, 1/4\}$ and $\{\eta_1, \eta_2\} = \{1/2, 2/3\}$.

Subsequently, by taking advantage of the additivity of expectation operation, we obtain the relation infra for the unconditional BLER $P_e\left(\mathbf{x}(t)\right)$:

$$\begin{aligned} &P_e\left(\mathbf{x}(t)\right) \approx \mathop{\mathbb{E}}_{\mathbf{H}}\left\{\sum_{\hat{\mathbf{x}}(\hat{t})\neq\mathbf{x}(t)} P_e\left(\mathbf{x}(t) \to \hat{\mathbf{x}}(\hat{t})|\mathbf{H}\right)\right\} \\ &= \sum_{\hat{\mathbf{x}}(\hat{t})\neq\mathbf{x}(t)} \mathop{\mathbb{E}}_{\mathbf{H}}\left\{P_e\left(\mathbf{x}(t) \to \hat{\mathbf{x}}(\hat{t})|\mathbf{H}\right)\right\} = \sum_{\hat{\mathbf{x}}(\hat{t})\neq\mathbf{x}(t)} P_e\left(\mathbf{x}(t) \to \hat{\mathbf{x}}(\hat{t})\right). \end{aligned} \tag{8}$$

Finally, with the assumption that all incoming bits are equiprobable, we can average $P_e\left(\mathbf{x}(t)\right)$ over all legitimate OFDM blocks $\mathbf{x}(t) \in \mathcal{X}$ and obtain the average BLER as

$$\bar{P}_e = \frac{1}{X}\sum_{\mathbf{x}(t)\in\mathcal{X}} P_e\left(\mathbf{x}(t)\right), \tag{9}$$

where $X = |\mathcal{X}| = 2^{\lfloor \log_2 \binom{N}{K} \rfloor} M^K$ is the number of all legitimate OFDM blocks.

2) Average BER: Similarly for average BER, we can approximate it simply by the union bound approach based on the unconditional PEP derived in (5), and obtain

$$\bar{P}_b = \frac{1}{XB}\sum_{\mathbf{x}(t)\in\mathcal{X}} \sum_{\hat{\mathbf{x}}(\hat{t})\neq\mathbf{x}(t)} P_e\left(\mathbf{x}(t) \to \hat{\mathbf{x}}(\hat{t})\right) \delta\left(\mathbf{x}(t) \to \hat{\mathbf{x}}(\hat{t})\right), \tag{10}$$

where $\delta\left(\mathbf{x}(t) \to \hat{\mathbf{x}}(\hat{t})\right)$ represents the number of bit errors for a single pairwise error event that $\mathbf{x}(t)$ is erroneously estimated to be $\hat{\mathbf{x}}(\hat{t})$.

IV. Numerical Results

To verify the proposed error performance analysis methodology based on Craig's formula and compare it with the conventional methodology relying on the exponential approximation of $Q(x)$, we carried out a set of simulations by Monte Carlo methods. Simulation results are presented and discussed in this section. For simplicity, we normalize the fading channel by setting $\mu = 1$ without loss of generality and utilize binary PSK (BPSK), i.e. $M = 2$ as well as Gray code for bit mapping. We take the analytical results generated by conventional methodology relying on the exponential approximation of Q-function as comparison benchmarks in our simulations. Simulation results pertaining to average BLER and BER are illustrated in Fig. 1 and Fig. 2, respectively. From both figures, we can observe that the analytical accuracy has been improved for both average BLER and BER, as the gaps between analytical and numerical results narrow in comparison with those provided by conventional methodology. Besides, it can also be found that the union bound assisted by PEP analysis yields a significant impact on error performance when P_t/N_0 is small.

$$
\begin{aligned}
P_e\left(\mathbf{x}(t) \rightarrow \hat{\mathbf{x}}(\hat{t})\right) &= \mathop{\mathbb{E}}_{\mathbf{H}}\left\{P_e\left(\mathbf{x}(t) \rightarrow \hat{\mathbf{x}}(\hat{t})|\mathbf{H}\right)\right\} = \frac{1}{\pi}\mathop{\mathbb{E}}_{\mathbf{H}}\left\{\int_0^{\frac{\pi}{2}}\prod_{n=1}^{N}\exp\left(-\frac{P_tG(n)\Delta(n,t,\hat{t})}{2KN_0\sin^2\theta}\right)\mathrm{d}\theta\right\} \\
&= \frac{1}{\pi}\underbrace{\int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty}}_{N-\text{fold}}\left(\int_0^{\frac{\pi}{2}}\prod_{n=1}^{N}\exp\left(-\frac{P_tG(n)\Delta(n,t,\hat{t})}{2KN_0\sin^2\theta}\right)\mathrm{d}\theta\right)\left(\prod_{n=1}^{N}f_g(G(n))\right)\mathrm{d}G(1)\mathrm{d}G(2)\ldots\mathrm{d}G(N) \\
&\overset{\text{(a)}}{=} \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left[\underbrace{\int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty}}_{N-\text{fold}}\left(\prod_{n=1}^{N}\exp\left(-\frac{P_tG(n)\Delta(n,t,\hat{t})}{2KN_0\sin^2\theta}\right)\right)\left(\prod_{n=1}^{N}f_g(G(n))\right)\mathrm{d}G(1)\mathrm{d}G(2)\ldots\mathrm{d}G(N)\right]\mathrm{d}\theta \qquad (5) \\
&\overset{\text{(b)}}{=} \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left[\prod_{n=1}^{N}\left(\int_0^{\infty}\exp\left(-\frac{P_tG(n)\Delta(n,t,\hat{t})}{2KN_0\sin^2\theta}\right)f_g(G(n))\mathrm{d}G(n)\right)\right]\mathrm{d}\theta = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left[\prod_{n=1}^{N}\left(1+\tau(n,t,\hat{t})\csc^2\theta\right)^{-1}\right]\mathrm{d}\theta \\
&\overset{\text{(c)}}{=} \frac{1}{\pi}\sum_{n=1}^{N}\left[\alpha(n,t,\hat{t})\int_0^{\frac{\pi}{2}}\left(1+\tau(n,t,\hat{t})\csc^2\theta\right)^{-1}\mathrm{d}\theta\right] = \frac{1}{2}\sum_{n=1}^{N}\left(\frac{\alpha(n,t,\hat{t})}{1+\tau(n,t,\hat{t})+\sqrt{\tau(n,t,\hat{t})(1+\tau(n,t,\hat{t}))}}\right)
\end{aligned}
$$

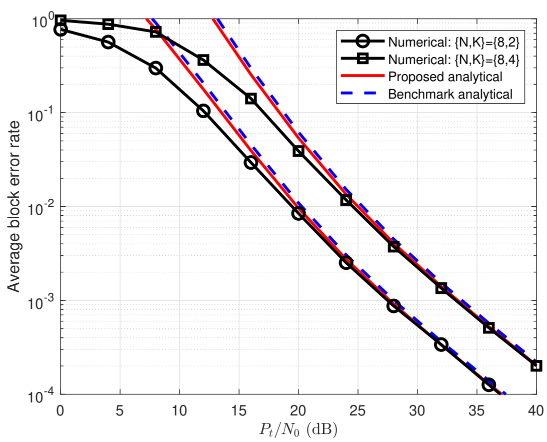

Fig. 1. Average BLER vs. ratio of transmit power to noise power P_t/N_0.

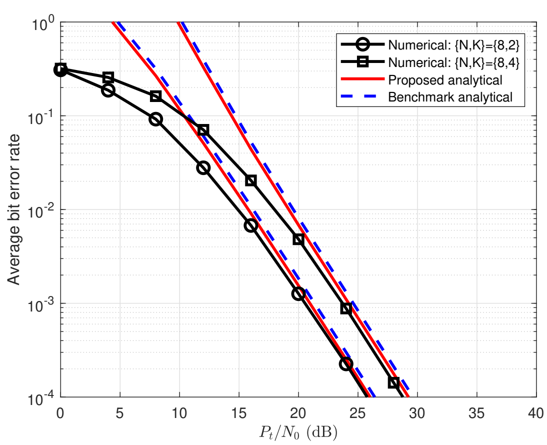

Fig. 2. Average BER vs. ratio of transmit power to noise power P_t/N_0.

V. Conclusion

In order to obtain more accurate analytical results pertaining to the error performance for OFDM-IM and explore the inherent mechanism of the error rate gaps between analytical and numerical results in existing works, we proposed a new error performance analysis based on Craig's formula for OFDM-IM. Therefore, the only approximation adopted to perform the error performance analysis of OFDM-IM is the union bound. We derived the average BLER and BER in closed form by the proposed analytical methodology and examined them by numerical simulations. Numerical results verified that the analytical accuracy is improved by the proposed methodology in comparison with the conventional methodology relying on the exponential approximation of Q-function. Moreover, the impacts of both approximations, i.e. the union bound and the exponential approximation of Q-function utilized in existing works for error performance analysis can be clarified.

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