Queuing Systems with Server Vacations

Vacation Policies

- Vacation startup rule:
  - exhaustive.
  - non-exhaustive.
- Vacation termination rule:
  - single vacation.
  - multiple vacation.
  - threshold policy.
- Vacation duration:
  - usually assumed to be i.i.d.

M/G/1 with Exhaustive Multiple i.i.d Vacations- Introduction

- The system goes to a vacation if there are no waiting customers.
- The system may take another vacation if there are no waiting customers upon the end of the previous vacation.
- The vacation duration is a random variable, V, with mean $\bar{V}$ and second moment $\overline{V^2}$.
- The residual vacation time $\overline{R_v} = \frac{\overline{V^2}}{2\bar{V}}$.

M/G/1 with Exhaustive Multiple i.i.d Vacations- Analysis

- $w_i = u(k)r_i + \big(1 - u(k)\big)v_i + \sum_{j=i-n}^{i-1} x_j$
- $\mathrm{u}(k) = \begin{cases} 1 & k = 1,2,3, \dots \\ 0 & otherwise \end{cases}$
- $\overline{W} = \rho\bar{R} + (1-\rho)\overline{R_v} + N_q\bar{x}$
- $\overline{W} = \frac{\rho}{1-\rho}\bar{R} + \overline{R_v}$, simplifying we get:
- $\overline{W} = \frac{\lambda\overline{x^2}}{2(1-\rho)} + \frac{\overline{V^2}}{2\overline{V}}$

M/G/1 with Exhaustive Multiple i.i.d Vacations- Performance Measures

- The response time:

$$\bar{T} = \bar{x} + \frac{\lambda \overline{x^2}}{2(1-\rho)} + \frac{\overline{V^2}}{2\bar{V}}$$

- The number of customers in the queue:

$$\overline{N_q} = \frac{\lambda^2 \overline{x^2}}{2(1-\rho)} + \lambda \frac{\overline{V^2}}{2\bar{V}}$$

- The number of customers in the system:

$$\bar{N} = \rho + \frac{\lambda^2 \overline{x^2}}{2(1-\rho)} + \lambda \frac{\overline{V^2}}{2\bar{V}}$$

Non-preemptive Priority M/G/1 Queue with Vacation.

- The average waiting time of class i:
  $\overline{W_i} = \overline{W_i^q} + \overline{W_i^a} + \overline{W_i^r} + \overline{W_i^v}$
  $\overline{W_i^v} = (1 - \sigma_M)\,\overline{R_v}$
- With the same M/G/1 analysis, we get:
- $\overline{W_i} = \dfrac{\sum_{j=1}^{M} \dfrac{\overline{\lambda_j}\overline{x_j^2}}{2} + (1-\sigma_M)\,\overline{R_v}}{(1-\sigma_{i-1})(1-\sigma_i)}$