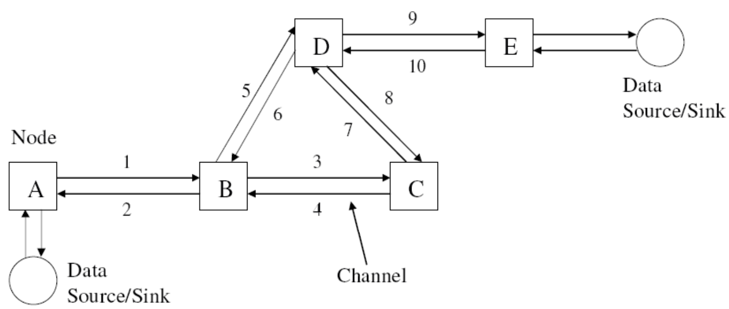
Applications to Computer Networks

Open Network Model

Store-an-Forward Packet-Switched Network

- Example



Assumptions for the Classical Model

- FCFS servers are used to model communication channels and data sources/sinks
- Packet processing time at switching node is negligible
- Kleinrock's independence assumption
  - each time a packet joins a queue in the network, its length is determined afresh from the p.d.f.

$$b(x) = \mu e^{-\mu x}, \quad x \geq 0$$

where $\frac{1}{\mu}$ = mean packet length

Assumptions for the Classical Model

- Unlimited buffer space
- No transmission error
- Path-oriented routing - a set of paths is defined for each source-destination node pair
- Packets are assumed to belong to different classes, each class corresponds to a source-destination node pair
  - total no. of classes = K
- Interarrival time of class k packets is exponentially distributed with parameter γ_k

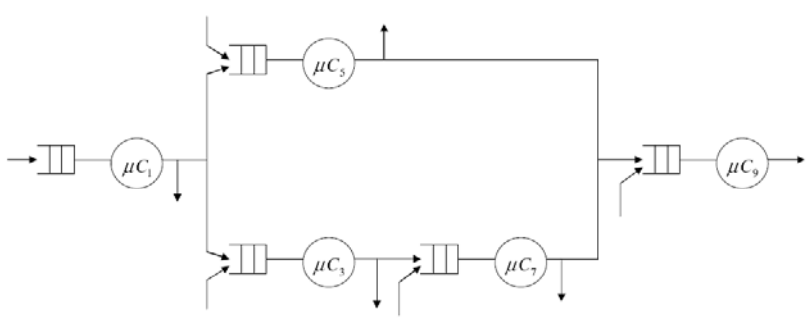
Notation and Definition

- C_i = capacity of channel i
  - service time at channel i (server i) is exponential with parameter μC_i
- Each path is characterized by an ordered set of channels

  π_k = set of channels in path k (or class k)

Open Network Model with Multiple Classes

- for simplicity, only odd numbered channels are shown



Open Network Model with Multiple Classes

- Suitable for "datagram networks"
- Analysis is based on the approach for Markovian queuing networks
  - provides results for end-to-end delay of each class
  - results are useful for network design:
    - Capacity assignment
    - Optimal routing
    - Topological design

Analytic Results

- Let λ_{ik} = mean total arrival rate of class k packets to channel i
- For path-oriented routing,

$$\lambda_{ik} = \begin{cases} \gamma_k & \text{if } i \in \pi_k, \\ 0 & \text{otherwise.} \end{cases}$$

Analytic Results

- Mean no. of class k packets at channel i

$$\overline{n}_{ik} = \frac{\rho_{ik}}{1-\rho_i} \qquad \text{where } \rho_{ik} = \frac{\lambda_{ik}}{\mu C_i}$$

- Mean no. of packets (from all classes) at channel i

$$\overline{n}_i = \sum_{k=1}^{K} \overline{n}_{ik} = \frac{\rho_i}{1-\rho_i} \qquad \text{where } \rho_i = \sum_{k=1}^{K} \rho_{ik}$$

Analytic Results

- Mean delay at channel i

$$t_{ik} = \frac{\overline{n}_{ik}}{\lambda_{ik}} = \frac{1}{\mu C_i(1-\rho_i)} \quad \text{(same for all classes)}$$

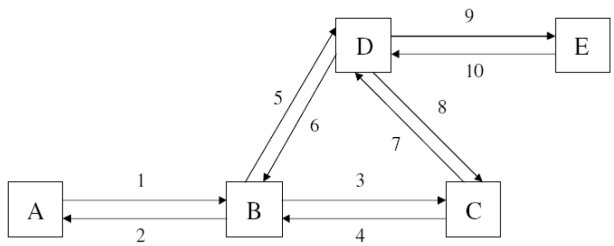
- Mean end-to-end delay

i. class k : $\overline{T}_k = \sum_{i\in\pi_k} \frac{1}{\mu C_i(1-\rho_i)}$

ii.over all classes :

$$\overline{T} = \sum_{k=1}^{K} \frac{\gamma_k}{\gamma} \overline{T}_k \qquad \text{where } \gamma = \sum_{k=1}^{K} \gamma_k$$

Shortest-Path Routing Example



Shortest-Path Routing Example

- One path (shortest path) for each source-destination node pair. The γ_k's are given by traffic matrix below (α is a parameter)

	A	B	C	D	E
A	0	α	α	3α	2α
B	α	0	4α	2α	α
C	α	4α	0	3α	α
D	3α	2α	3α	0	2α
E	2α	α	α	2α	0

Shortest-Path Routing Example

- Calculation of λ_{ik} and λ_i

	Channel i				
Class	1	3	5	7	9
1: A → B	α				
2: A → C	α	α			
3: A → D	3α		3α		
4: A → E	2α		2α		2α
5: B → C		4α			
6: B → D			2α		
7: B → E			α		α
8: C → D				3α	
9: C → E				α	α
10: D → E					2α
$\Sigma = \lambda_i$	7α	5α	8α	4α	6α

End-to-End Delay Results

- Suppose $\mu C_i = 1$ for all i
- Consider end-to-end delay from node A to node E (class 4)

$$T_{A,E} = T_4 = \frac{1}{1-7\alpha} + \frac{1}{1-8\alpha} + \frac{1}{1-6\alpha}$$

- *Note*: propagation delay needs to be added

Alternate Routing Example

- One path (shortest path) for each source-destination node pair except $A \rightarrow E$
  - from node A to node E, two paths are defined
    {1, 5, 9} – class 4, $\gamma_4 = 1.5\alpha$
    {1, 3, 7, 9} – class 4', $\gamma_{4'} = 0.5\alpha$

Alternate Routing Example

- Calculation of λ_{ik} and λ_i

Class	Channel i: 1	3	5	7	9
1: A → B	α				
2: A → C	α	α			
3: A → D	3α		3α		
4: A → E(1)	1.5α		1.5α		1.5α
4': A → E(2)	0.5α	0.5α		0.5α	0.5α
5: B → C		4α			
6: B → D			2α		
7: B → E			α		α
8: C → D				3α	
9: C → E				α	α
10: D → E					2α
$\Sigma = \lambda_i$	7α	5.5α	7.5α	4.5α	6α

End-to-End Delay Results

- Mean end-to-end delay from node A to node E (class 4)
  - path {1, 5, 9} is used

$$\overline{T}_4 = \frac{1}{1-7\alpha} + \frac{1}{1-7.5\alpha} + \frac{1}{1-6\alpha}$$

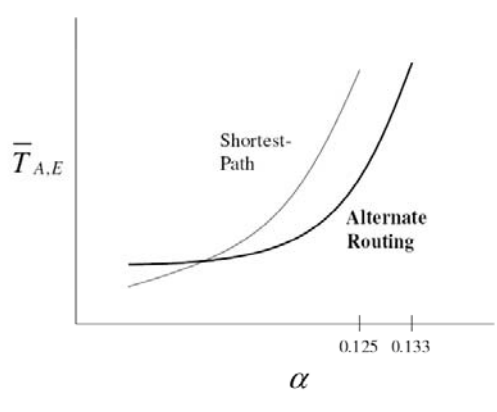
  - path {1, 3, 7, 9} is used

$$\overline{T}_{4'} = \frac{1}{1-7\alpha} + \frac{1}{1-5.5\alpha} + \frac{1}{1-4.5\alpha} + \frac{1}{1-6\alpha}$$

  - average over two paths

$$\overline{T}_{A,E} = 0.75\overline{T}_4 + 0.25\overline{T}_{4'}$$

Typical Behavior



Observations

- At light load (small α), shortest path routing is better for $A \to E$ traffic because there is not much interference
- At large α, splitting $A \to E$ traffic into 2 paths is better for $A \to E$ traffic because the load on the network is more balanced