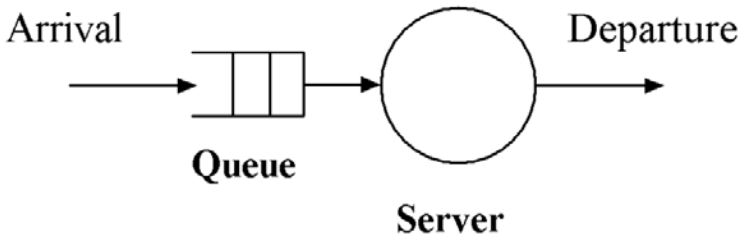
Analytic Modeling

Deterministic Model

Infinite Population Model



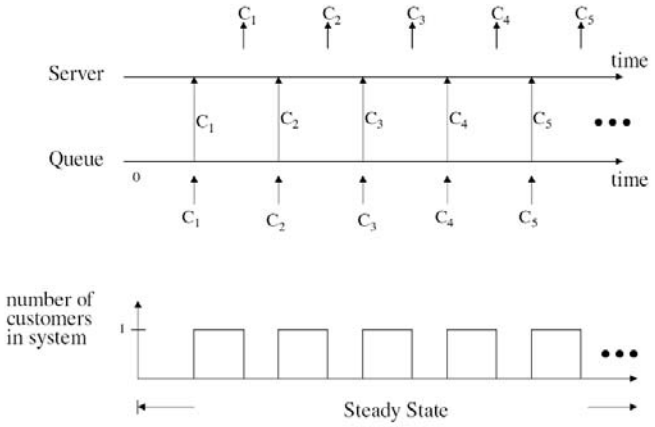
Analytic Results

- Let interarrival time = y (with probability 1.0)
- Let service time = x (with probability 1.0)
- If $x < y$, waiting time = 0 and utilization = x/y at steady state (independent of initial condition)
- If $x > y$, queue becomes unbound as time progresses (steady state behavior is not defined)
- Special case, $x = y$, utilization = 1.0 and waiting time at steady state is dependent of initial condition

Example 1

- Interarrival time = 3 seconds
- Service time = 2 seconds
- Initial condition: system is empty

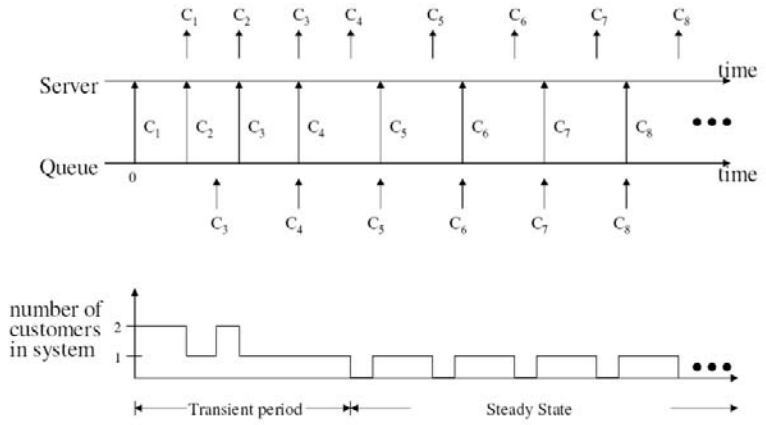
Timing Diagram



Example 2

- Interarrival time = 3 seconds
- Service time = 2 seconds
- Initial condition: 2 customers are already in system and the server starts serving one of them

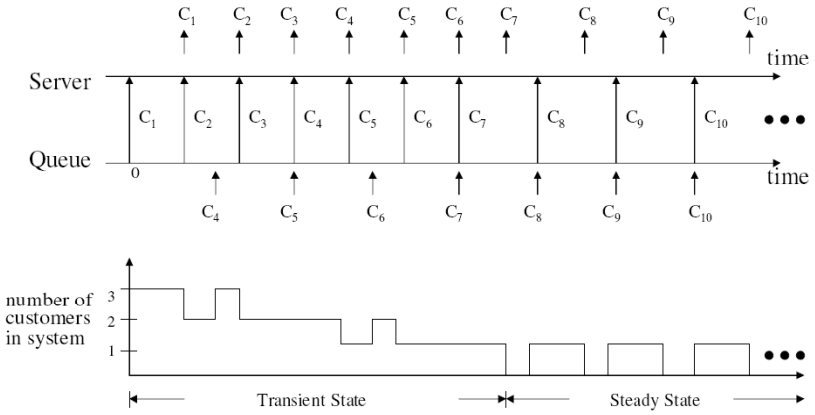
Timing Diagram



Example 3

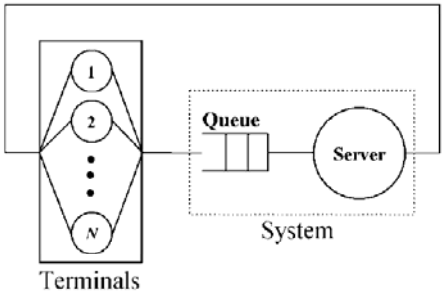
- Interarrival time = 3 seconds
- Service time = 2 seconds
- Initial condition: 3 customers are already in system and the server starts serving one of them

Timing Diagram



Finite Population Model

- Single server case

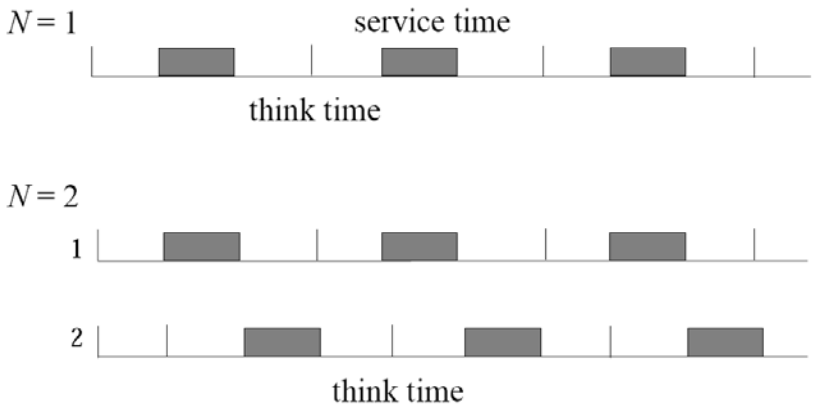


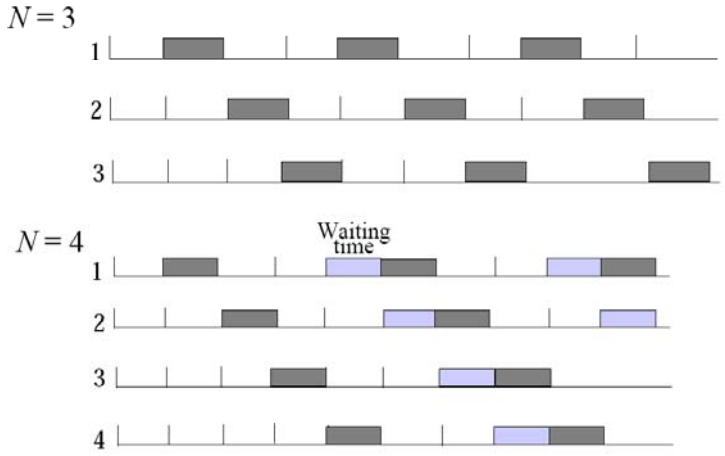
- Let think time = h
- Let service time = x

Example

- Think time = 2
- Service time = 1
- Number of users = N, users are indexed by i
- Initial condition: user i (i = 1, 2, …, N) submits the first request at time i
- FCFS discipline

Timing Diagram





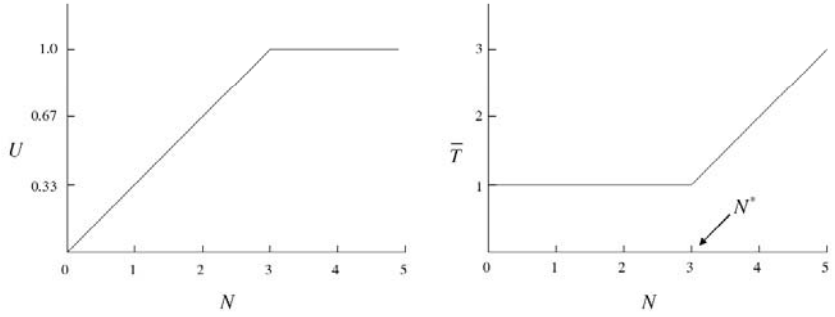
Analytic Results –Utilization Factor U

- Assume h is divisible by x
- Let $N^* = \dfrac{(x+h)}{x}$
- At steady state, $U = \begin{cases} N\dfrac{x}{x+h} & 1 \le N \le N^* \\ 1 & N \ge N^* \end{cases}$

Analytic Results –Response Time T

- Assume h is divisible by x
- Let $N^* = \dfrac{(x+h)}{x}$
- At steady state, $T = \begin{cases} x & 1 \le N \le N^* \\ Nx - h & N \ge N^* \end{cases}$

Performance Results



$h = 2, x = 1, N^* = 3$

- Exact analytic results are available
- Performance measures are expressed as functions of input parameters
- Utilization is 100% when $N \ge N^*$
- Response time increases linearly with N when $N > N^*$