Assignment 3, Advanced Markovian Queues

Due Date is: Oct. 19th 2021

Question 1, About M/M/1/K queue

Consider an M/M/1/n model for a router. The packets arrival process is a Poisson process with rate λ. The size of packet (unit of size is bit, which is approximately continuous since packet size is usually round several hundred kilo-bits) is exponentially distributed with mean 1. The router's processing rate is a constant μ (unit is bit/s). The service discipline for packets is FCFS. Answer the following questions. p.s, write down the detailed calculation process.

a) Calculate the utilization rate of this system, i.e., the probability that the router is busy.

b) Calculate the loss probability of this system, i.e., the ratio of packets that are rejected by the router.

c) Calculate the average routing time for each packet, i.e., the customer average response time of this queueing system.

d) Suppose $n = 5$, $\lambda = 0.3$, and $\mu = 1$, is it better to double the buffer size or the processing speed?

e) Suppose $n = 5$, $\lambda = 0.8$, and $\mu = 1$, what's the answer for the question above?

f) Discuss the insights for system design that you obtain from the two different situations above. The same budget for system updating, is it used to increase the buffer size or processor speed?

Question 2, MATLAB Simulation

Use MATLAB to code a discrete-event simulation program for the following two types of queues.
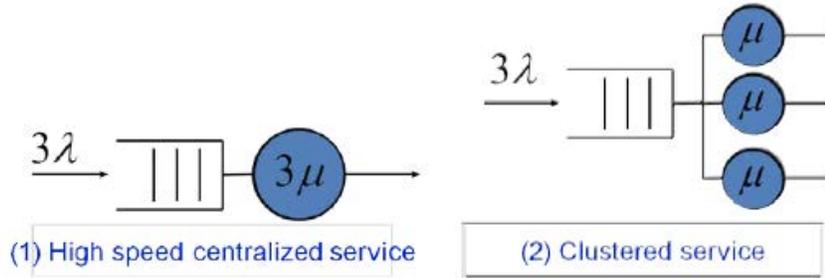
Compare their average number of customers, average response time, and average waiting time, under different variability of arrival processes. (Coefficient of variability of arrival processes is, Cv = 0.5, 1, 3, respectively. Use Erlang-k, Exponential, and Hyper-exponential distribution to simulate the arrival processes, respectively. Note, the parameters of Erlang-k and Hyper-exponential should be carefully chosen to satisfy the value of Cv. The effective arrival rate is $\lambda = 1$ and the service rate is $\mu = 2$.) List the comparison values on a table and do necessary analysis and discussion. Hint: the Erlang-k distribution has a mean k/α, and coefficient of variability $(1/k)0.5$. In order to simulate 3λ arrival process with Cv = 0.5, you should use an Erlang-4 distribution with $\lambda = 12$, where γ is the mean rate of each stage service in Erlang distribution. For a hyper-exponential distribution with selection probability α, its mean is

$$E(X) = \sum_{i=1}^{k} \frac{\alpha_i}{\mu_i}$$

and its variance is

$$\sigma^2(X) = 2 \sum_{i=1}^{k} \frac{\alpha_i}{\mu_i^2} - \left(\sum_{i=1}^{k} \frac{\alpha_i}{\mu_i} \right)^2$$

choose proper α and μ_i to make its mean equals $1/\mu = 1/2$ and Cv=3. One approximation is that α=(3/7, 4/7) and $\mu_1 = 1$, $\mu_2 = 8$, the corresponding mean is 0.5 and Cv is 2.5. You can choose other parameters with a better approximation.

The MATLAB code along with the README file should be submitted electronically in a single zip file.

Question 3, MATLAB Simulation

Use MATLAB to code a discrete-event simulation program to compare the performance of M/M/1 queue with different service disciplines, under different variability of arrival processes. Let the Coefficient of variability of arrival processes (Cv = 0.5, 1, 3, respectively). The performance metrics include average number of customers, average response time, and average waiting time. The service discipline includes:

a) FCFS: first come first serve;

b) PR-LCFS: preemptive-resume last come first serve.

c) OPTIONAL: FCFS with service vacations

List the comparison values on a table and do necessary analysis and discussion.

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