Assignment 2, Elementary Queueing Theory

Due Date is: Sept. 28th 2021

Question 1, About comparison of different types of M/M/c queues

Consider three kinds of queueing systems:

1. an M/M/1 system with arrival rate 3λ and service rate 3μ;
2. an M/M/3 system with one sharing queue, the arrival rate is 3λ and service rate of each server is μ;
3. an M/M/3 system with 3 independent queues, that is, 3 identical M/M/1 queue with arrival rate λ and service rate μ;

Compare their dynamics and answer the following questions.

a) Calculate the average number of customers L for these 3 systems.

b) Calculate the customer average waiting time T_w for these 3 systems.

c) Calculate the customer average response time T_r for these 3 systems.

d) Suppose $\mu = 2\lambda$. Compare these results above and get the hints on how to design the service facility for a website: Think about different strategies for server deployment, single high-performance server v.s. multiple servers as a cluster v.s. multiple servers distributed deployed.

Question 2

Customer A arrives at a service facility with 3 servers at time $t = 0$ and finds all the servers are busy and 4 other customers waiting for service in the queue. The service discipline is FCFS and we assume there is not any further customer arrivals after $t = 0$. Service times are assumed to be mutually independent, identical, exponentially distributed random variables, each with mean duration $1/\mu$.

a) Define the system state of this queueing system and draw the state transition diagram.

b) Calculate the expected response time of customer A.

c) Calculate the average length of period from time $t = 0$ to the time that there is at least one server idle.

d) Let Y be the order of completion of service of customer A: that is, $Y = k$ if A is the k-th customer to complete service after $t = 0$. Calculate the probability $P[Y = k]$, $k = 1, 2, \cdots, 8$.

e) Find the probability that customer A completes service after the time that the customer immediately ahead of him finishes the service in the queue.

f) Let w be the amount of time customer A waits for service. Calculate the probability density function of w, $p[w = x]$.

Question 3, About M/M/1 queue with priority

Consider an M/M/1 queue for 2 types of customers. Type 1 customer has higher priority. The service discipline is preemptive resume with priority. That is, when type 1 customer arrives and type 2 customer is in service, the service of type 2 customer will be interrupted and type 1 customer gets service instantly. When the system does not have type 1 customer, the interrupted type 2 customer will continue its service from the last interruption point.

The arrival rate of type 1 and 2 customers are λ_1 and λ_2, respectively. The service rate for type 1 and 2 customers are μ_1 and μ_2, respectively.

a) Write the balance equation of this queueing system to solve the steady state distribution. (No need to give the analytical solution, just list the equations)

b) With the Little's law, use mean value analysis method to obtain the mean sojourn time and average number of customers of type 1 and 2 customers, respectively.

Question 4, MATLAB Simulation

Use MATLAB to code a discrete-event simulation program for an M/M/1 queue

a) Plot a typical sample path curve of a busy period; show the events of customer arrival and departure on the picture.

b) Obtain the statistics of the main performance metrics of this queue, including the average number of customers, average response time, average waiting time, and throughput. Compare them with the theoretical formulas.

c) Plot a curve of the average queue length/average waiting time w.r.t. service rate.

The MATLAB code along with the README file should be submitted electronically in a single zip file.